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DETERMINATION OF OPTIMAL PING STRATEGY FOR RANDOM ACTIVE SONAR SEARCH IN A COUNTERDETECTION ENVIRONMENT

by

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Optimization of Optimal Ping Strategy for Random Active Sonar Search in a Counterdetection Environment

by

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ABSTRACT

This thesis is an analysis of the one-on-one ASW search problem using a random active search strategy in an environment that favors the target's counterdetection ability. The objective is to determine an optimum ping strategy by simulation of the definite-range problem, approximation by an analytical model and use of empirical regression techniques.

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I. INTRODUCTION

A. THE THESIS OBJECTIVES

This thesis documents the analysis of one-on-one ASW encounters between a surface searcher using active sonar and an evasive target submarine. The analysis is based on data generated by a computer simulation of the relative motion of the two adversaries over time. The specific objective of this analysis is to prescribe a strategy for selecting a searcher ping interval which maximizes the probability of detection in an environment which favors the target's counter-detection ability.

B. THE SCENARIO

A single surface ship is assigned to search for a submarine target of interest using active sonar within a region several hundred thousand square miles in area. The acoustic environment is considered homogeneous throughout the area. Thus for any particular case, the sonar range is considered a constant.

1. The Searcher's Tactic

The searcher's tactic is to move through the area at a set speed, changing course randomly at times described by an exponential distribution of mean $1/\theta$. The use of the exponential distribution for this purpose seems tactically prudent because of its memoryless property. The searcher

pings at times selected at random from some distribution. By this tactic, the exact time between successive pings is made unpredictable.

2. The Target's Tactic

The target is patrolling the area of interest at a set speed, changing course randomly at times also selected from an exponential distribution, not necessarily the same as, but independent of, that of the searcher. The target is capable of counterdetecting the searcher's transmissions at ranges greater than the searcher's detection ranges. It is assumed that the target has no method of detecting and locating the searcher other than by passive detection of the searcher's transmissions. If the target does counterdetect the searcher outside the searcher's detection range, he sprints away from the searcher radially at a speed greater than that of the searcher. The duration of this sprint is a tactical decision made by the target, based on what he considers a "safe" range, and can only be estimated by the searcher.

II. MODEL DESCRIPTION

It is commonly assumed that the time, T_0, required for a randomly moving searcher and target to first come within some relatively small range of one another is distributed exponentially with a mean that is a function of that range, the searcher's and target's speeds, and the size of the area in which they are confined [Ref. 1]. Intuitively, it would seem if the searcher selects a ping strategy that maximizes the probability of detection, given that the target is within counterdetection range, that strategy tends to minimize the expected total time, T, spent searching in the area for the target. With this in mind, a model of the search problem after the realization of T_0 is described below.

A. THE EVENT DISK

In the model used for simulation, the event disk, i.e., the region within an event circle of radius C, the counterdetection range, represents the area in which any interaction between the target and searcher must occur. Concentric with the event disk is the searcher's detection disk of radius D. The event disk is centered on that opponent with the higher speed. Figure 2.1 illustrates the case for which the searcher is at a higher speed than the target, but the labeling is completely arbitrary because of symmetry. If the target is at

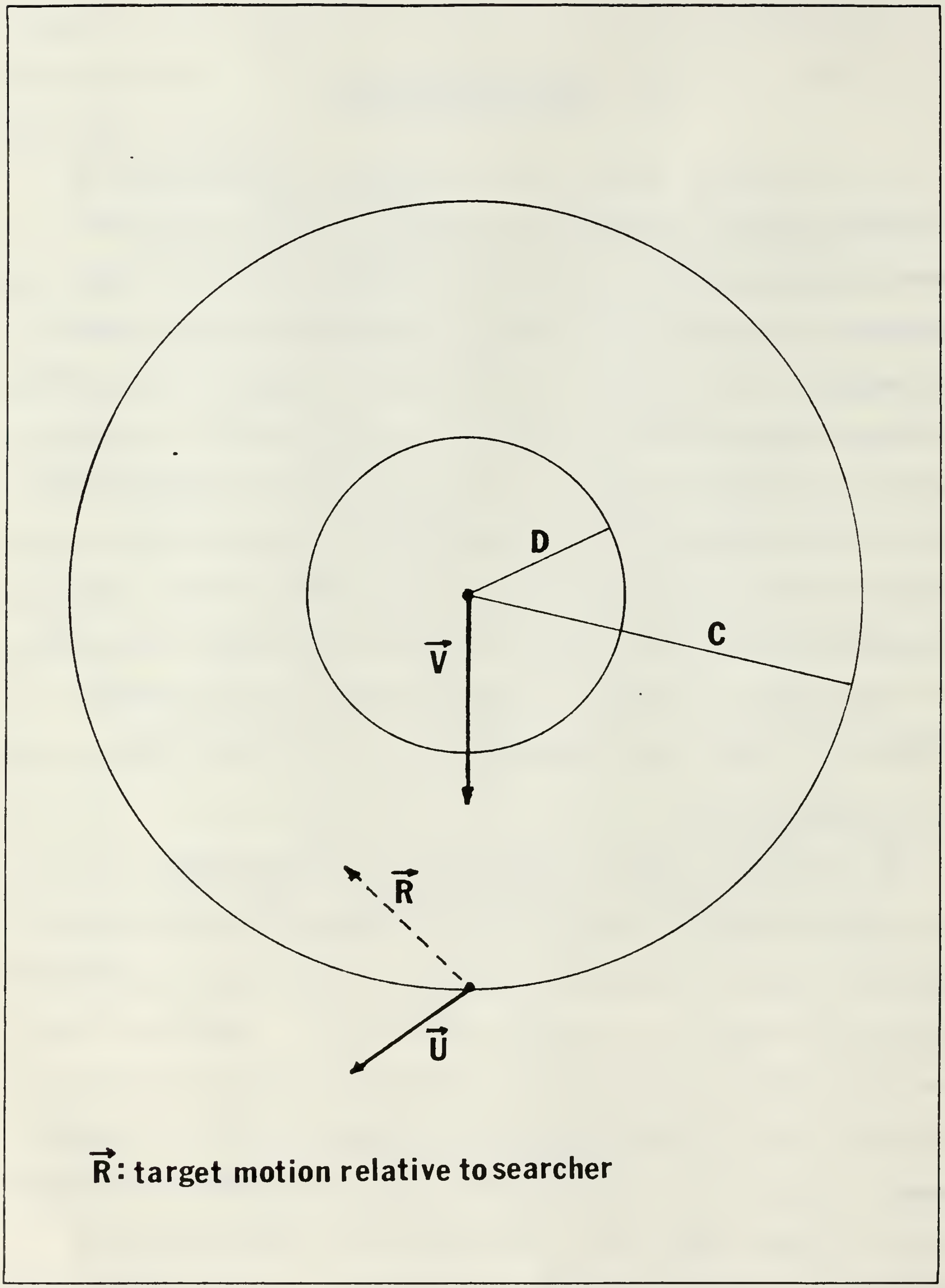

Figure 2.1 The Event Disk.

the higher speed, he is placed in the center. All relative relationships remain the same.

B. THE EXPECTED SEARCH TIME

Once the target has entered the event disk, one of three events must occur:

(1) The target departs the event disk before the searcher's first ping, by virtue of the relative motion between the two.

(2) The target sprints out of the event disk as a result of being located in the counterdetection zone but not in the searcher's detection zone when the searcher pings.

(3) The target is detected as a result of being located in the detection zone when the searcher pings. This event completes the search.

If Events (1) or (2) occur, then there is a possibility that, eventually, the target will again enter the event disk. Therefore, once the target has entered the event disk for the first time, the remainder of the search can be thought of as a series of cycles during which the target is either detected or not detected. This suggests the use of the geometric distribution to describe the process. If the search requires N such cycles, then (N-1) of the cycles must have resulted in no detection occurring. Therefore, if P is the probability of detection, the probability that the search requires n cycles is

$$P\ (N = n) \quad = \quad (1-P)^{n-1}P \qquad (2.1)$$

Determining, $E[T]$, the expected total time for completing the search requires that two additional variables be defined. Let

$$T_c(i) \quad i = 1,2,3,\ldots,n$$

be a sequence of independent, identically distributed random variables describing the cycle time for the i^{th} cycle and let T_d be the time required for the target to enter the detection zone and be detected, given he is located on the perimeter of the event disk. Then

$$T = T_0 + \sum_{i=1}^{N-1} T_c(i) + T_d \tag{2.2}$$

and

$$E[T|N] = E[T_0] + (n+1)E[T_c] + E[T_d] \tag{2.3}$$

Removing the condition on n results in the following expression:

$$E[T] = E\{E[T|N]\} = E[T_0] + E[T_c] \sum_{n=1}^{\infty} (n-1)(1-P)^{n-1}P + E[T_d] \tag{2.4}$$

The summation term is easily collapsed. Let

$$S = \sum_{n=1}^{\infty} (n-1)(1-P)^{n-1}P = \sum_{n=0}^{\infty} n(1-P)^{n}P$$

$$= 0 + (1-P)P + 2(1-P)^2P + \ldots \qquad (2.5)$$

Then

$$(1-P)S = (1-P)^2P + 2(1-P)^3P + 3(1-P)^4P + \ldots \qquad (2.6)$$

Subtracting Equation (2.6) from Equation (2.5) yields

$$PS = P(1+P)[1 + (1+P) + (1+P)^2 + \ldots] \qquad (2.7)$$

The sum inside the brackets is a geometric series which converges to 1/P. Therefore

$$E[T] = E[T_0] + (\frac{1}{P} - 1)E[T_c] + E[T_d] \qquad (2.8)$$

It can be seen that maximizing the probability of detection, P, will aid in minimizing the expected total search time. It is for this reason that this thesis concentrates on the problem within the event disk; that is, finding a ping rate which maximizes P.

C. CRITICAL ASSUMPTIONS

Several assumptions have been made to simplify the model and analysis of the generated data. In addition to those stated previously, the following also apply:

(1) The occurrence of detection and counterdetection events is determined using a definite-range or "cookie cutter" model.

(2) The target and searcher have negligible baffle areas.

(3) Detection and counterdetection ranges are not degraded with speed.

(4) There is no convergence zone considered, nor are there any gaps in the event circle.

(5) The target is strictly evasive.

(6) Counterdetection range as used in the model should be considered the target's minimum desired range to the searcher. It is assumed this range is at least twice the detection range.

D. THE REQUIREMENT FOR A PING STRATEGY

The assumed existence of a definite counterdetection range greater than the searcher's detection range requires that the searcher have a well-defined minimum interval between any two active pings. This minimum interval is:

$$I_{min} = \frac{C - D}{U + V} \tag{2.9}$$

where C is the counterdetection range, D the detection range, and the denominator is the sum of the two speeds. This is merely the time required for the target to move from the perimeter of the event disk to the detection zone at the maximum attainable relative speed.

There is also a maximum practical ping interval that is not as well defined. That is, if the searcher pings very infrequently he loses the opportunity for detection because the target may transit in and then out of the detection zone between pings. This "maximum" should depend upon the size of the detection zone, and the relative motion between the searcher and target. It is because of the relative motion aspects that this maximum practical interval cannot be defined as easily as the minimum. The existence of a minimum ping interval below which the probability of detection is zero, and a "maximum" ping interval beyond which the probability of detection is small, implies that the probability of detection reaches a maximum between these two extremes. This maximum should be a function of the ranges and speeds specific to each particular case. Therefore the first step is to determine a ping strategy as it depends upon the independent variables. This will be done by simulation, approximate analytical modeling, and a blending of the two by an empirical regression technique.

E. DATA GENERATION

The equation for probability of detection if both searcher and target remain on constant courses and speeds is complex but can be solved using polar coordinates and some trigonometry. However, in the problem at hand, both relative speed and its direction change randomly. This urges the use of simulation to generate data on relative courses and positions.

The simulation program for the definite range problem (included in Appendix C for informational purposes) supplies as output the number of detections, the number of counterdetections, and the number of times the target departs the event disk before the first ping, for a given ping interval, detection range, counterdetection range, searcher speed, and target speed. The frequency of course changes, the searcher course, and the target course are determined by random number generators. All relative motion is placed on the target using trigonometric arguments with the searcher remaining at the coordinate origin.

Because each interaction begins with the range between the searcher and target decreasing to C, it follows that the initial relative velocity must be directed into the event disk. To accomplish this in the simulation, the first relative velocity vector was determined by assuming that the searcher's speed component directly toward the target was just greater than the target's speed. After the initial leg, all motion was unconstrained. So after several course changes, the effect of the initial leg is "forgotten" by the process.

An alternate method, not used in this thesis, would be to let the target's initial relative angle on the bow be selected from a cosine distribution. That is,

$$P(\text{Relative AOB} \leq \phi) = \frac{1}{2}\int_{-\pi/2}^{\phi} \cos\theta \, d\theta$$

$$= \frac{1}{2}(\sin\phi + 1), \quad -\pi/2 \leq \phi \leq \pi/2 \qquad (2.10)$$

This procedure is suggested by Koopman's observation that when searching for a stationary target, the bearing of initial detection will have a cosine distribution [Ref. 1].

It is not known which of these methods is more correct. It may be possible to derive an exact expression for the joint distribution of the target's relative speed and course at detection, but this was not accomplished here.

Once the time for the first ping is reached, the replication is stopped. The range between the two is calculated and compared to the values for C and D to determine which of the possible events has occurred. The outcome is stored and the whole process is repeated for the desired number of trials. The data is then analyzed to obtain estimates of the probability of detection and other relevant quantities, such as expected times within and without the event disk.

III. DETERMINATION OF THE REGRESSION MODEL

Once the data has been generated, regression may be used to determine an empirical predictive formula for the optimum ping rate. Armed with only intuitive hypotheses, the search for the "best" functional form of the input variables would be difficult. So it is desirable to find some theoretical guidelines for selecting candidate explanatory variables to use in the regression model.

A. USE OF THE VON NEUMANN FUNCTION

The theoretical model selected for use is one for energy transmission and return when the target's motion is modeled by a diffusion process. Define the searcher's location as the origin on a Cartesian plane, and define the target's location at time t, as [X(t),Y(t)]. Let the target undergo Brownian motion so that its location at time t is described by

$$\begin{aligned} X(t) &\sim N(X(0,\sigma^2 t) \\ Y(t) &\sim N(Y(0),\sigma^2 t) \end{aligned} \qquad (3.1)$$

and let the searcher's probability of detection, for a ping at time t be

$$P(t|X(t),Y(t)) = e^{-(X(t)^2+Y(t)^2)/\delta^2}$$

$$= e^{-\frac{1}{2}(R(t)^2)/\delta^2} \qquad (3.2)$$

where R(t) is the range from the searcher to the target and δ^2 is a, thus far, unspecified constant. The constant δ in the detection function (Equation (3.2)) plays a role analogous to that of the detection disk radius, D. In particular, δ is that range where the probability of detection is $e^{-1/2} \approx 0.607$.

Removing the condition on position by integrating over all values of X(t) and Y(t) results in the following expression for P(t):

$$P(t) = \int_{-\infty}^{\infty}\int e^{-\frac{1}{2}(X(t)^2)/\delta^2} \frac{1}{\sqrt{2\pi\sigma^2 t}} e^{-\frac{1}{2}[((X(t)-X(0))^2)/\sigma^2 t]} dX$$

$$e^{-\frac{1}{2}((Y(t)^2)/\delta^2} \frac{1}{\sqrt{2\pi\sigma^2 t}} e^{-\frac{1}{2}[((Y(t)-Y(0))^2)/\sigma^2 t]} dY$$

$$= e^{-\frac{1}{2}[(R(0)^2/\delta^2)/(1+\sigma^2 t/\delta^2)]} \frac{1}{(1+\sigma^2 t/\delta^2)} \qquad (3.3)$$

Using the natural log function to linearize P(t), and differentiating with respect to t yields

$$\frac{d\,\ell n P(t)}{dt} = \frac{\sigma^2/\delta^2}{1+\sigma^2 t/\delta^2}\left[\frac{1}{2}\frac{R^2}{\delta^2} - (1+\frac{\sigma^2 t}{\delta^2})\right] \qquad (3.4)$$

Setting the derivative equal to 0 results in the following expression for T*, the optimum ping interval:

$$T^* = \frac{\frac{1}{2}\left(\frac{R(0)^2}{\delta^2}\right) - 1}{\left(\frac{\sigma}{\delta}\right)^2} \tag{3.5}$$

Therefore

$$T^* = 0 \quad \text{if} \quad \frac{1}{2}\,\frac{R(0)^2}{\delta^2} \leq 1$$

and

$$T^* = \frac{\frac{1}{2}\left(\frac{R(0)}{\delta}\right)^2 - 1}{\left(\frac{\sigma}{\delta}\right)^2} \quad \text{otherwise} \tag{3.6}$$

Referring to Equation (3.2), suppose energy from a ping transmitted from location (0,0) reaches the target with probability

$$e^{-\frac{1}{2}\left(\frac{R(t)}{\delta_o}\right)^2}, \tag{3.7}$$

and the probability of the signal returning to the searcher given it reached the target is

$$e^{-\frac{1}{2}\left(\frac{R(t)}{\delta_i}\right)^2}. \tag{3.8}$$

Then the probability of the searcher detecting the target becomes

$$P(\text{detection } R(t)) = e^{-\frac{1}{2}\left(\frac{R(t)}{\delta_o}\right)^2} e^{-\frac{1}{2}\left(\frac{R(t)}{\delta_i}\right)^2}$$

$$= e^{-\frac{1}{2}R(t)^2\left[\frac{1}{\delta_o^2}+\frac{1}{\delta_i^2}\right]}$$

$$= e^{-\frac{1}{2}\left(\frac{R(t)}{\delta}\right)^2} \qquad (3.9)$$

which is also a Von Neumann detection function.

The intention at this point is to use the form of Equation (3.5) to develop an approximation for the optimal ping interval T^*_{DR} for the definite range problem. This is accomplished by setting R(0) in Equation (3.5) equal to the counterdetection range, C, and recognizing that the parameter δ is a "characteristic range" for the Von Neumann function. So

$$\frac{1}{\delta^2} = \left[\frac{1}{\delta_o^2} + \frac{1}{\delta_i^2}\right] = \left[\frac{1}{R^2} + \frac{1}{r^2}\right] \qquad (3.10)$$

where R and r are the ranges associated with the outbound and inbound acoustic paths.

Substituting into Equation (3.5) yields

$$T^*_{DR} = \frac{\frac{1}{2}C^2\left(\frac{1}{C^2}+\frac{1}{r^2}\right) - 1}{\sigma^2\left(\frac{1}{C^2}+\frac{1}{r^2}\right)} \qquad (3.11)$$

Letting r now be the active detection range, D, results in the following expression:

$$T^*_{DR} = [\frac{C^2}{2\sigma^2}][\frac{1/C^2 + 1/D^2 - 2/C^2}{1/C^2 + 1/D^2}]$$

$$= [\frac{C^2}{2\sigma^2}][\frac{1/D^2 - 1/C^2}{1/D^2 + 1/C^2}]$$

$$= [\frac{C^2}{2\sigma^2}][\frac{C^2/D^2 - 1}{C^2/D^2 + 1}] \qquad (3.12)$$

B. THE EXPLANATORY VARIABLES

It remains to replace the diffusion constant σ with appropriate random tour model parameters u, v, and λ. Lambda is the parameter for the exponential distribution describing the minimum time between course changes for either of the adversaries. If σ^2 is a diffusion rate describing the relative motion of the two then

$$\sigma^2 = \frac{U^2 + V^2}{\lambda} \qquad (3.13)$$

is dimensionally correct and has some theoretical appeal. Specifically, an unconstrained two dimensional random tour with constant speed V and rate of course change λ_V can be approximated for large times by a diffusion process with constant V^2/λ_V [Ref. 2]. When two particles are simultaneously

conducting random tours, then the composite random tour in relative space has a rate of course change:

$$\lambda = \lambda_u + \lambda_v \tag{3.14}$$

and a random speed S_R. If the angle between the V and U velocity vectors is uniformly distributed between 0 and 2π radians then the expected value of S_R^2 is:

$$E[S_R^2] = \frac{1}{2\pi}\int_0^{2\pi} (U^2 + V^2 - 2\ UV \cos\theta)d\theta$$

$$= U^2 + V^2 \tag{3.15}$$

Using $(U^2 + V^2)$ as a representative squared speed for the composite random tour and λ for the rate of course changes yields Equation (3.13).

Equation (3.12) is related to, but probably unequal to, the optimum ping interval for the definite range law model that is being simulated. In order to better adapt the Von Neumann model results to the definite range law data, one can redefine T^*_{DR} as

$$T^*_{DR} = \beta_1 \left[\frac{\lambda C^2}{(U^2 + V^2)}\right]^{\beta_2} \left[\frac{C^2/D^2 - 1}{C^2/D^2 + 1}\right]^{\beta_3} \tag{3.16}$$

and determine the parameters β_1, β_2, and β_3 by regression, using as explanatory variables, the quantities

$$\left[\frac{\lambda C^2}{(U^2+V^2)}\right] \quad \text{and} \quad \left[\frac{C^2/D^2-1}{C^2/D^2+1}\right] .$$

It is acknowledged that Equation (3.16) is probably not the "best" definite range law extrapolation of the Von Neumann result of Equation (3.5). In particular, setting δ_i equal to the detection range D seems suspect since the active detection process involves two-way propagation and δ_i considers primarily the return path. Nonetheless, Equation (3.16) was tested as a candidate regression model and, as the next section documents, it performed quite well.

IV. THE RESULTS

A total of 180 cases were run using 4 different speed combinations, detection ranges from 3 to 16 miles, and several counterdetection ranges between 20 and 40 miles. The probability of detection was measured at each of 50 different ping intervals, beginning at the practical minimum and stepped in 0.05 hour increments. A total of 500 trials were run at each ping interval. The observed T* was that ping interval at which the maximum probability of detection occurred for each case. In the event of a tie, the earlier time was used. Figure 4.1 illustrates the dependence of the probabilities of detection, counterdetection, and departure on the value of ping interval. As expected, the probability of detection is 0 until the ping interval is greater than (C-D)/(U+V), increases to a maximum and slowly decreases to a small positive value. The probability of departure increases with ping interval, while the probability of counterdetection decreases.

A. THE EMPIRICAL PREDICTION FORMULA

Performing linear regression on the data, using the explanatory variables discussed previously, produced the following equation for predicting T^*_{DR}:

$$T^*_{DR} = 0.74\left[\frac{\lambda C^2}{(U^2+V^2)}\right]^{0.661}\left[\frac{C^2/D^2-1}{C^2/D^2+1}\right]^{0.173} \qquad (4.1)$$

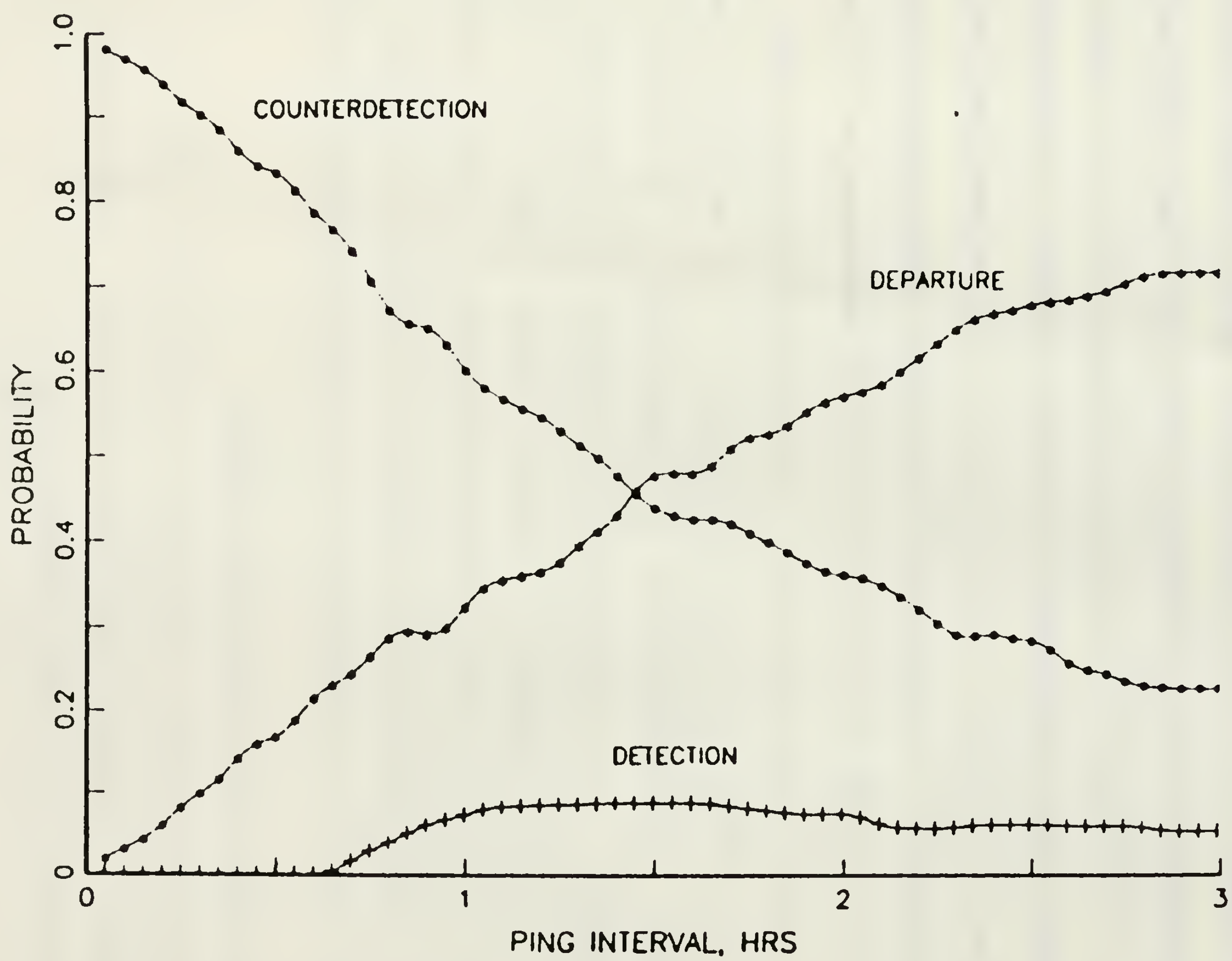

Figure 4.1 Time Dependency of Event Probabilities.

This formula explained 89% of the total variation in the data and all coefficients were significant at the 0.99 level using Student's t statistic.

Tables (1)-(4) of Appendix A list the input variables, observed T* and predicted T* for each case. It is immediately apparent that the probability of detection at the predicted value of T* is consistently less than or equal to that at the observed value of T*. This is because of the initial relatively crude method used to choose the observed T* for each case. The values for the difference in the probabilities, a measure of the prediction validity, are relatively small. Figure 4.2 is a scatter plot of the probability of detection at the predicted T* versus that at the observed T*. The ideal plot would be a straight line of slope 1 through the origin. The least squares fitted line through the data has a slope of 0.94 and intercept of -0.01. Many of the points away from the diagonal can be explained by examination of the raw data. Often, the predicted and observed optimum ping intervals are within 0.10 hours of one another, but the variance of the sampled binomial distribution causes the two detection probabilities to appear farther apart than might actually be the case. It should be mentioned that smoothing half of the raw data using running medians followed by running averages (Hanning) and using the same regression model did not alter the coefficients of the prediction formula significantly. It did, as expected, tend to decrease the difference between

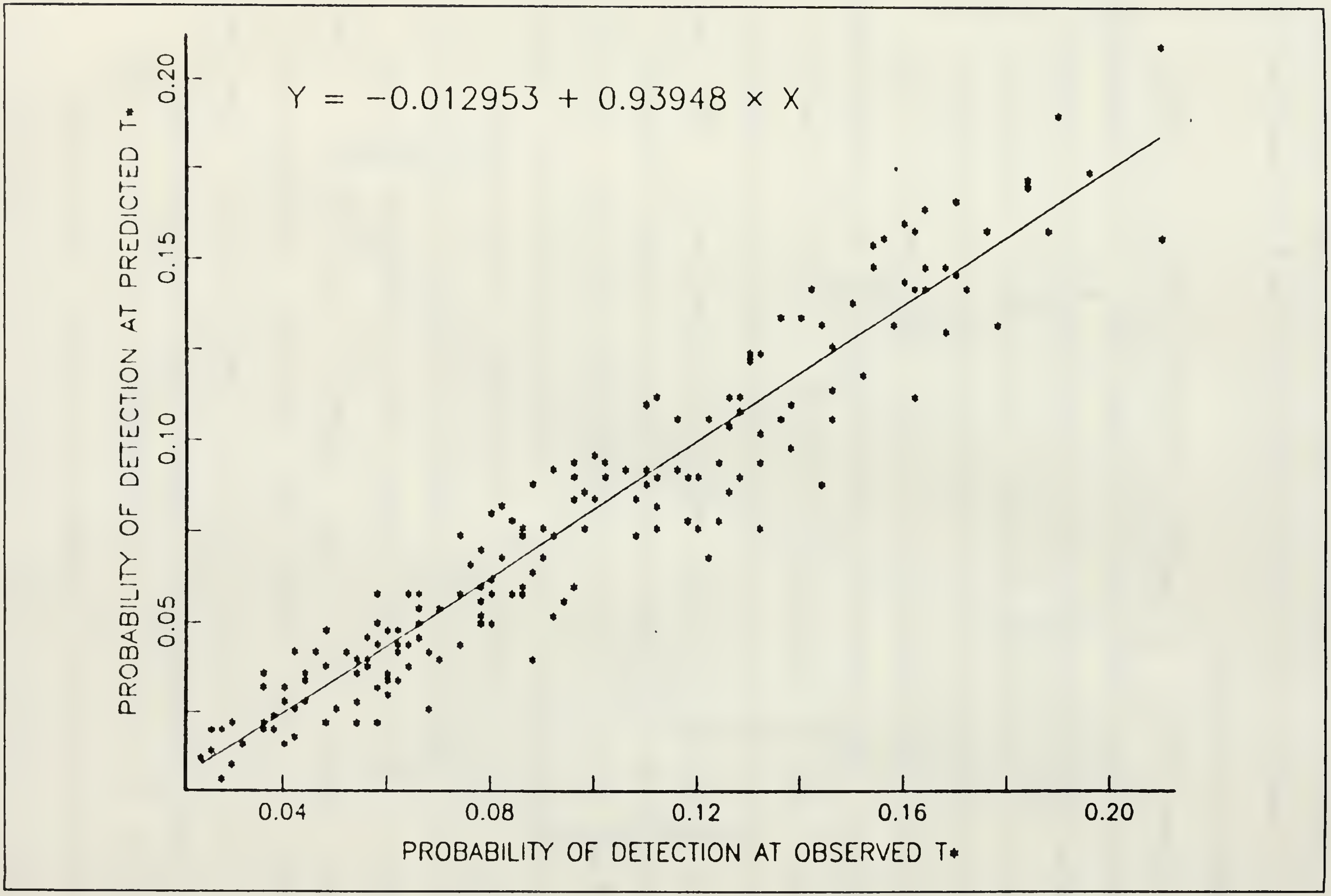

Figure 4.2 Using the Predicted T*.

the value for probability of detection at the predicted T*
and that at the observed T*.

B. CONCLUSIONS

The most significant finding is that it is possible to determine an optimum ping strategy within the limitations of the model. If a searcher were tasked with such a search, he would not ever want to ping more frequently than the minimum ping interval (C-D)/(U+V). This assumes the searcher has a good estimate of the counterdetection range and speed of the target. On the average, he would want to ping at a rate that is slightly less than that prescribed by the prediction.

There are two reasons for this: (1) the probability of detection decreases more slowly to the right of T* than it increases at the left; and (2) any deviation to the right of T* merely increases the probability of the departure event occurring. The departure event is considered less detrimental to the search effort than the counterdetection event, for which the probability of occurrence steadily decreases.

The Von Neumann function seems well-suited as a theoretical foundation for determination of an optimum ping strategy. This suggests that research into its use in a more realistic model of active sonar search might prove valuable in predictions of this sort. The extremely sharp cut off of the definite range law is certainly an artificiality.

APPENDIX A

TABLES

TABLE 1

COMPARISON OF OBSERVED AND PREDICTED T*

SEARCHER SPEED: 15 KTS TARGET SPEED: 5 KTS
LAMBDA: 1.5/HR

C(NM.)	D(NM.)	OBSERVED T*	OBSERVED P(T*)	PREDICTED T*	PREDICTED P(T*)
20	3	1.75	0.030	1.30	0.022
	4	1.30	0.058	1.30	0.058
	5	1.55	0.070	1.30	0.040
	6	1.35	0.094	1.30	0.056
	7	1.45	0.146	1.25	0.126
	8	1.25	0.156	1.25	0.156
22	4	1.65	0.042	1.50	0.018
	5	1.90	0.056	1.45	0.040
	6	1.20	0.076	1.45	0.066
	7	1.25	0.122	1.45	0.068
	8	1.40	0.130	1.45	0.124
	9	1.75	0.162	1.40	0.158
25	4	1.80	0.028	1.75	0.020
	5	1.35	0.058	1.75	0.022
	6	1.70	0.060	1.75	0.048
	7	1.80	0.092	1.70	0.052
	8	1.85	0.100	1.70	0.096
	9	1.85	0.128	1.70	0.112
	10	1.75	0.140	1.70	0.134
27	4	2.05	0.028	1.95	0.006
	5	2.60	0.040	1.95	0.028
	6	2.00	0.054	1.95	0.028
	7	2.35	0.064	1.90	0.058
	8	2.00	0.084	1.90	0.078
	9	1.55	0.098	1.90	0.086
	10	2.10	0.132	1.90	0.076
	11	1.95	0.136	1.85	0.106
30	6	2.90	0.042	2.25	0.026
	7	2.10	0.060	2.20	0.030
	8	1.90	0.066	2.20	0.050
	9	2.40	0.078	2.20	0.070
	10	2.00	0.092	2.15	0.074
	11	2.35	0.146	2.15	0.106
	12	1.85	0.126	2.15	0.112
	13	2.20	0.162	2.10	0.112
	14	2.00	0.164	2.10	0.142
40	8	3.70	0.044	3.25	0.034
	9	3.30	0.052	3.25	0.042
	10	3.10	0.046	3.25	0.042
	11	2.80	0.064	3.20	0.038
	12	3.65	0.066	3.20	0.046
	13	3.05	0.080	3.20	0.062
	14	3.05	0.082	3.15	0.068
	15	2.85	0.112	3.15	0.076
	16	2.35	0.112	3.10	0.082

TABLE 2

COMPARISON OF OBSERVED AND PREDICTED T*

SEARCHER SPEED: 18 KTS
TARGET SPEED: 7 KTS
LAMBDA: 1.5/HR

C(NM.)	D(NM.)	OBSERVED		PREDICTED	
		T*	P(T*)	T*	P(T*)
20	3	1.05	0.026	1.00	0.020
	4	0.90	0.044	1.00	0.034
	5	1.00	0.074	1.00	0.074
	6	1.00	0.092	1.00	0.092
	7	1.00	0.130	0.95	0.122
	8	0.95	0.190	0.95	0.190
22	4	1.30	0.040	1.15	0.028
	5	1.25	0.054	1.15	0.040
	6	1.15	0.078	1.10	0.060
	7	1.35	0.098	1.10	0.076
	8	1.00	0.122	1.10	0.106
	9	1.25	0.178	1.10	0.132
25	4	1.30	0.032	1.35	0.016
	5	1.40	0.054	1.35	0.022
	6	1.50	0.062	1.35	0.044
	7	1.40	0.086	1.30	0.076
	8	1.60	0.096	1.30	0.084
	9	1.55	0.138	1.30	0.110
	10	1.25	0.168	1.30	0.148
27	4	1.30	0.024	1.50	0.012
	5	1.40	0.040	1.50	0.032
	6	2.00	0.060	1.50	0.034
	7	1.20	0.080	1.45	0.050
	8	1.70	0.096	1.45	0.090
	9	1.50	0.112	1.45	0.090
	10	1.75	0.132	1.45	0.094
	11	1.35	0.160	1.40	0.144
30	6	1.80	0.040	1.70	0.016
	7	2.00	0.056	1.70	0.038
	8	1.80	0.078	1.70	0.056
	9	1.70	0.082	1.70	0.082
	10	1.60	0.110	1.65	0.088
	11	2.15	0.128	1.65	0.108
	12	1.70	0.154	1.65	0.148
	13	1.65	0.168	1.60	0.130
	14	1.50	0.196	1.60	0.174
40	8	2.50	0.042	2.50	0.042
	9	2.65	0.048	2.50	0.022
	10	2.45	0.058	2.50	0.050
	11	2.35	0.068	2.45	0.042
	12	2.20	0.078	2.45	0.052
	13	2.45	0.088	2.45	0.088
	14	2.15	0.100	2.40	0.084
	15	3.25	0.108	2.40	0.074
	16	2.20	0.120	2.40	0.090

TABLE 3

COMPARISON OF OBSERVED AND PREDICTED T*

SEARCHER SPEED: 20 KTS
TARGET SPEED: 10 KTS
LAMBDA: 1.5/HR

C(NM.)	D(NM.)	OBSERVED T*	OBSERVED P(T*)	PREDICTED T*	PREDICTED P(T*)
20	3	0.75	0.036	0.85	0.022
	4	1.00	0.074	0.80	0.044
	5	0.80	0.080	0.80	0.080
	6	1.00	0.144	0.80	0.088
	7	0.80	0.154	0.80	0.154
	8	0.85	0.170	0.80	0.166
22	4	0.80	0.048	0.95	0.038
	5	0.85	0.080	0.95	0.058
	6	0.85	0.082	0.90	0.082
	7	0.95	0.118	0.90	0.090
	8	1.00	0.144	0.90	0.132
	9	0.85	0.164	0.90	0.164
25	4	0.95	0.032	1.10	0.016
	5	0.95	0.058	1.10	0.032
	6	1.15	0.064	1.10	0.044
	7	1.30	0.086	1.10	0.058
	8	0.95	0.124	1.10	0.078
	9	0.90	0.128	1.05	0.090
	10	0.90	0.158	1.05	0.132
27	4	1.10	0.026	1.25	0.014
	5	1.10	0.038	1.20	0.024
	6	1.00	0.062	1.20	0.042
	7	1.10	0.088	1.20	0.040
	8	1.10	0.086	1.20	0.074
	9	1.25	0.120	1.20	0.076
	10	1.40	0.126	1.20	0.104
	11	1.20	0.162	1.15	0.142
30	6	1.20	0.040	1.40	0.028
	7	1.80	0.062	1.40	0.034
	8	1.30	0.078	1.40	0.050
	9	1.35	0.090	1.40	0.068
	10	1.45	0.108	1.35	0.084
	11	1.50	0.132	1.35	0.102
	12	1.30	0.164	1.35	0.148
	13	1.25	0.176	1.35	0.158
	14	1.25	0.188	1.30	0.158
40	8	1.55	0.038	2.05	0.020
	9	1.70	0.054	2.05	0.036
	10	1.70	0.068	2.05	0.026
	11	2.20	0.062	2.05	0.048
	12	2.10	0.086	2.00	0.060
	13	2.20	0.096	2.00	0.094
	14	2.30	0.110	2.00	0.092
	15	1.90	0.116	2.00	0.092
	16	1.85	0.138	2.00	0.110

TABLE 4

COMPARISON OF OBSERVED AND PREDICTED T*

SEARCHER SPEED: 22 KTS
TARGET SPEED: 12 KTS
LAMBDA: 1.5/HR

C(NM.)	D(NM.)	OBSERVED T*	OBSERVED P(T*)	PREDICTED T*	PREDICTED P(T*)
20	3	0.70	0.036	0.70	0.036
	4	0.60	0.056	0.70	0.046
	5	0.75	0.096	0.70	0.084
	6	0.75	0.118	0.70	0.078
	7	0.80	0.146	0.70	0.114
	8	0.75	0.184	0.70	0.172
22	4	0.70	0.044	0.80	0.028
	5	0.65	0.078	0.80	0.050
	6	0.70	0.090	0.80	0.076
	7	0.70	0.124	0.80	0.094
	8	0.75	0.152	0.80	0.118
	9	0.85	0.184	0.75	0.170
25	4	1.20	0.036	0.95	0.020
	5	0.90	0.060	0.95	0.036
	6	1.10	0.066	0.95	0.058
	7	0.95	0.110	0.95	0.110
	8	0.95	0.112	0.95	0.112
	9	0.90	0.142	0.90	0.142
	10	0.90	0.210	0.90	0.210
27	4	0.85	0.030	1.05	0.010
	5	1.05	0.048	1.05	0.048
	6	1.05	0.058	1.05	0.058
	7	0.95	0.096	1.05	0.060
	8	0.95	0.102	1.05	0.094
	9	1.00	0.160	1.00	0.160
	10	0.95	0.136	1.00	0.134
	11	0.95	0.170	1.00	0.146
30	6	1.35	0.044	1.20	0.036
	7	1.10	0.058	1.20	0.044
	8	0.90	0.066	1.20	0.054
	9	1.00	0.102	1.20	0.090
	10	1.00	0.116	1.20	0.106
	11	1.10	0.132	1.20	0.124
	12	1.25	0.150	1.20	0.138
	12	1.00	0.172	1.15	0.142
	14	1.05	0.210	1.15	0.156
40	8	1.25	0.036	1.75	0.032
	9	2.45	0.050	1.75	0.026
	10	1.95	0.070	1.75	0.054
	11	1.45	0.074	1.75	0.058
	12	2.30	0.084	1.75	0.058
	13	1.65	0.088	1.75	0.064
	14	1.35	0.106	1.70	0.092
	15	1.45	0.126	1.70	0.086
	16	1.60	0.138	1.70	0.098

APPENDIX B

LIST OF DATA SOURCE VARIABLES

CRANGE - The target's counterdetection range.
CX ----- Coordinate origin.
DRANGE - The searcher's detection range.
DSEED -- The seed for uniform random number generator, GGUBS.
DTIME -- Time remaining on leg until a ping is due.
EM ----- The mean of the exponential distribution of minimum time between course changes by either searcher or target.
ESEED -- The seed for exponential random number generator, GGEXN.
GGUBS -- The IMSL subroutine for generating Uniform (0,1) random numbers.
GGEXN -- The IMSL subroutine for generating Exponential random numbers.
NDSETS - The number of data sets.
NLEG --- Number of motion leg on current trial, used to constrain sear--cher's initial course.
PCDET -- The probability of counterdetection.
PDET --- The probability of detection.
PEXIT -- The probability of departure.
PIMAX -- The ping interval at which PDET reached a maximum for current data set.
PING --- The ping interval for current trial.
PLIM --- Maximum PDET that has occurred thus far on current data set.
PTIME--- Variable used in deciding if a ping is due on current leg.
RAND --- The current Uniform (0,1) random number.
REXP --- The current Exponential random number.
SCRS --- The searcher's current course.
SECT --- The constraint boundary on the searcher's initial course.
TCRS --- The target's current course.
TCDCT--- The number of counterdetections at current ping interval.
TDCT --- The number of detections at current ping interval.
TEXIT--- The number of target departures before the first ping.
TIME --- Current time.
TRIAL--- Trial counter.
TSTEP -- The increment for increasing the ping interval
TX ----- The target's X-coordinate.
TY ----- The target's Y-coordinate.
U ------ The target's speed.
V ------ The searcher's speed.

APPENDIX C

THE DATA SOURCE PROGRAM LISTING

```
$JOB
C$OPTIONS TIME=(15)
      REAL RAND(1),REXP(1)
      DOUBLE PRECISION DSEED,ESEED,PI,SCRS,TCRS,TX,TY,RANGE
      PI=ARCOS(-1.0)
      TSTEP=.05
      READ,NDSET,DSEED,ESEED
      DO 200 NC=1,NDSET
      READ,PING,V,U,CRANGE,DRANGE,EM
      PLIM=0.0
      TLAMB=1/EM
      WRITE(6,450)NC
      WRITE(6,500)V,DRANGE
      WRITE(6,550)U,CRANGE
      WRITE(6,560)TLAMB
      WRITE(6,570)
      DO 100 I=1,50
      TDCT=0.0
      TCDCT=0.0
      TRIAL=0.0
      SECT=ARCOS(U/V)
      CX=0.0
   10 TY=CX
      TX=CX+CRANGE
      TIME=0.0
      PTIME=0.0
      NLEG=0
   20 CALL GGUBS(DSEED,1,RAND)
      SCRS=RAND(1)*2.0*PI
      IF(NLEG.EQ.0)SCRS=-SECT+RAND(1)*2.0*SECT
      CALL GGUBS(DSEED,1,RAND)
      TCRS=RAND(1)*2.0*PI
   30 CALL GGEXN(ESEED,EM,1,REXP)
      TLEG=REXP(1)
      TIME=PTIME+TLEG
      IF(TIME.GE.PING)THEN
        DTIME=PING-PTIME
        TY=TY-(DTIME*V*DSIN(SCRS))+(DTIME*U*DSIN(TCRS))
        TX=TX-(DTIME*V*DCOS(SCRS))+(DTIME*U*DCOS(TCRS))
        RANGE=DSQRT(TX**2+TY**2)
        IF(RANGE.LE.DRANGE)TDCT=TDCT+1.0
        IF(RANGE.GT.DRANGE.AND.RANGE.LE.CRANGE)TCDCT=TCDCT+1.0
        TRIAL=TRIAL+1.0
        GO TO 40
      ELSE
        TY=TY-(TLEG*V*DSIN(SCRS))+(TLEG*U*DSIN(TCRS))
```

```
      TX=TX-(TLEG*V*DCOS(SCRS))+(TLEG*U*DCOS(TCRS))
      RANGE=DSQRT(TX**2+TY**2)
      NLEG=NLEG+1
      PTIME=TIME
      IF(RANGE.GT.CRANGE)THEN
        TRIAL=TRIAL+1.0
        GO TO 40
      ELSE
        GO TO 20
      END IF
    END IF
 40 IF(TRIAL.EQ.500.0)THEN
      GO TO 50
    ELSE
      GO TO 10
    END IF
 50 TEXIT=TRIAL-TDCT-TCDCT
    PDET=TDCT/TRIAL
    PCDET=TCDCT/TRIAL
    PEXIT=TEXIT/TRIAL
    WRITE(6,600)PING,TDCT,TCDCT,TEXIT,PDET,PCDET,PEXIT
    IF(PDET.GT.PLIM)THEN
    PLIM=PDET
    PIMAX=PING
    END IF
    PING=PING+TSTEP
100 CONTINUE
    WRITE(6,650)
    WRITE(6,700)PLIM,PIMAX
    WRITE(6,750)
200 CONTINUE
450 FORMAT('1','CASE :',I3)
500 FORMAT('0','SEARCHER SPD: ',F4.1,' KTS',5X,'DETECTION RANGE: ',
   *F4.1,' NM')
550 FORMAT(1X,'TARGET SPD: ',F6.1,' KTS',5X,'COUNTERDETECTION RANGE: '
   *,F4.1,' NM')
560 FORMAT(1X,'LAMBDA: ',F4.2,' PER HOUR')
570 FORMAT('0','PING INT.',3X,'# DET',3X,'# CDET',3X,'# EXIT',3X,
   *'% DET',3X,'% CDET',3X,'% EXIT')
600 FORMAT(2X,F5.2,6X,F5.1,2X,F7.1,2X,F7.1,3X,3(F5.3,4X))
650 FORMAT('0')
700 FORMAT('0','MAX PROB. OF DET: ',F5.3,' OCCURRED USING PING INT: '
   *,F4.2,' HR.')
750 FORMAT(1X,'BASED ON 500 TRIALS AT EACH PING INTERVAL')
    STOP
    END
$ENTRY
```

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